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The Transformation Problem: Erroneous Arguments of Tugan-Baranovsky, Bortkiewicz and Steedman

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Abstract: This paper analyzes erroneous arguments of Tugan-Baranovsky, Bortkiewicz, and Steedman which are still used by the critics of the theory of commodity values transformation into prices of production, described by Marx in Volume III of *Capital*. The methods of the original and final transformation of values into prices of production are proposed, which correspond to Marx's theory and confirm the validity of his arguments. The author has proposed a method for the inverse transformation of prices into value, which allows one to determine where the surplus value is actually produced.

Keywords: Marxism, labor theory of value, transformation problem, postulates invariance, real wages, total social product, the inverse transformation prices into values.

JEL classification: B12, B14, B24, B51, D46, E11

Introduction

Economists have been trying to solve the *transformation problem* since 1885, when F. Engels, the editor of volumes II and III of K. Marx's *Capital*, in the introduction to volume II, asked the economists before the publication of volume III:

"... How an equal average rate of profit can and must come about, not only without a violation of the law of value, but rather on the very basis of it." (Marx, 1997 [1885]: 23).

Nine years later, in the introduction to volume III of the *Capital*, Engels was forced to upset, to one degree or another, the economists who responded to this proposal (V. Lexis, J. Wolf, A. Loria, G. C. Stiebeling). He gave a moderately positive assessment only to the studies of the economists of the Marx school — P. Fireman and C. Schmidt — who only “which has accomplished something” to solve the problem (Marx, 1998 [1894]: 11). From volume III of the *Capital*, it became clear that the solution to the problem presented by Marx consisted in the fact that, in order to establish a uniform rate of profit in all the spheres of production, the total amount of surplus value is to be redistributed among these separate spheres of production in such a way that two macroeconomic identity conditions, which subsequently received the name of invariance postulates, hold:

“... The sum of the profits in all spheres of production must equal the sum of the surplus values, and the sum of the prices of production of the total social product equal the sum of its value.” (Marx, 1998 [1894]: 172).

However, the described transformation procedure is not sufficient, since it leaves without change the cost of production of goods that depend on the prices of production through elements of constant and variable capital. Marx took into account this circumstance, as he wrote:

“We had originally assumed that the cost price of a commodity equalled the *value* of the commodities consumed in its production. But for the buyer the price of production of a specific commodity is its cost price, and may thus pass as cost price into the prices of other commodities. Since the price of production may differ

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from the value of a commodity, it follows that the cost price of a commodity containing this price of production of another commodity may also stand above or below that portion of its total value derived from the value of the means of production consumed by it. It is necessary to remember this modified significance of the cost price, and to bear in mind that there is always the possibility of an error if the cost price of a commodity in any particular sphere is identified with the value of the means of production consumed by it. Our present analysis does not necessitate a closer examination of this point.” (Marx, 1998 [1894]: 164).

One of the first economists to consider this point in detail was the prominent economist, mathematician and statistician, representative of the “Russian-German school”, Ladislaus Josephovich Bortkiewicz (1868-1931). Ironically, his article written in 1907, which is fundamental to set the problem of the transformation of values into production prices, was published in Russian for the first time only in 2011 (see Bortkiewicz, 2011), although Bortkiewicz is mentioned by Vladimir Ilyich Lenin, as a critic of Marx, in his work *Karl Marx*, printed in 1915 (Lenin, 1964 [1914-5]): 43-91). In particular, in this article Lenin writes:

“In this way, the well-known and indisputable fact of the divergence between prices and values and of the equalization of profits is fully explained by Marx on the basis of the law of value, since the sum total of values of all commodities coincides with the sum total of prices. However, the equating of (social) value to prices (individual) prices does not take place simply and directly, but in a very complex way. It is quite natural that in a society of separate producers of commodities, who are united only by the market, a conformity to law can be only an average, social, mass manifestation, with individual deviations in either direction mutually compensating one another.” (Lenin, 1964 [1914-5]): 66).

The first attempt to consider the modification of the value for the production costs can be considered the works of Wolfgang Mühlpfordt, relative to 1893 and 1895 and remained unknown until 1987, when the researchers M. Howard and J. King made them public (Howard and King, 1987). According to their statement, Mühlpfordt proposed a solution similar to the one that was given several years later by Bortkiewicz. However, it is now known that Mühlpfordt tried to use instead of the postulate “the sum of the prices of production of the total social product = the sum of its value” the alternative postulate of invariance “the sum of new values = the sum of their prices” (see Quaas, 1994: 2).

Currently, most well-known economists believe that the problem transformation does not have a rigorous mathematical solution. The insolvability of the transformational problem, in the development of which many well-known economists took part, is considered the main proof of the impossibility of transforming values into the production prices, and is the evidence of the allegedly theoretical errors of Marx, which affect the logic of the *Capital*. Moreover, on the basis of the works of the neo-Ricardian Ian Steedman (Steedman, 1977), which showed that the relative production prices and the general rate of profit can be determined without using labor costs, it is concluded that the effort of the Marx’s value theory can be generally discarded without compromising the results of the economic analysis.

In this article we show that such a conclusion is wrong, since the problem of transformation is solved not only without the destruction of the categories of “value” and “surplus value”, but just on their basis.

Main results of the study

Bortkiewicz proposed his method of solving the problem using the scheme of general reproduction by Mikhail Ivanovich Tugan-Baranovsky (Tugan, 2003 [1905]), who, in turn, modified the scheme of simple reproduction by Marx from volume II of the *Capital*. These changes slightly affected the differentiation of the organic composition of capital, but caused a radical change in the composition of the departments of subdivision II of the annual production of goods. In Marx, this subdivision is divided into two large departments:

“a) Articles of consumption, which enter into the consumption of the working class, and, to the extent that they are necessities of life — even if frequently different in quality and value from those of the labourers — also form a portion of the consumption of the capitalist class. For our purposes we may call this entire sub-division consumer *necessities*, regardless of whether such a product as tobacco is really a consumer necessity from the physiological point of view. It suffices that it is habitually such.

b) Articles of *luxury*, which enter into the consumption of only the capitalist class and can therefore be exchanged only for spent surplus value, which never falls to the share of the labourer.” (Marx, 1997 [1885]: 402).

Therefore, the scheme of simple reproduction in Marx takes the following form:

Marx's reproduction scheme (see Marx, 1997 [1885]: 402-410)

I. Production of means		
of production.....	$4000c_1 + 1000v_1 + 1000m_1 = 6000w_1$	
IIa. Production of necessities.....	$1600c_2 + 400v_2 + 400m_2 = 2400w_2$	(1)
IIb. Production of luxuries.....	$400c_3 + 100v_3 + 100m_3 = 600w_3$	
Total annual commodity product...	$6000C + 1500V + 1500M = 9000W$	

As we can see, Marx's scheme (1) satisfies the inequalities $2400w_2 > 1500V$ and $600w_3 < 1500M$, respecting the general equality $w_2 + w_3 = V + M$.

Tugan-Baranovsky's 'sub-division IIa (or II)' produces only the workers' consumption goods (wage goods), and the capitalists' consumption goods are, obviously, in the 'sub-division II-b (or III)', where luxury goods are mainly produced.

Tugan-Baranovsky reproduction scheme in value (Tugan, 2003 [1905]: 157)

I. Production of means of production...	$225c_1 + 90v_1 + 60m_1 = 375w_1$	
II. Production of workers' consumption		
goods (wage goods).....	$100c_2 + 120v_2 + 80m_2 = 300w_2$	(2)
III. Production of capitalists'		
consumption goods (luxury goods)...	$50c_3 + 90v_3 + 60m_3 = 200w_3$	
Total annual commodity product.....	$375C + 300V + 200M = 875W$	

In scheme (2), instead of the aforementioned inequalities, the equalities $300w_2 = 300V$ and $200w_3 = 200V$ are satisfied, always respecting the general equality $w_2 + w_3 = V + M$.

It was scheme (2), expressed in values, that L. von Bortkiewicz used as the basis for illustrating the mechanism for transforming values into production prices, replacing the previous natural units for measuring the product values (i.e. thousands of working years) with monetary units. Note that in scheme (2) the newly created value is proportional to the cost of living labour in all sectors, since $(v + m) : v = \text{const} = 1.667$. In addition, we will show that in this case the value prices (i.e. prices taken equal to values) are directly proportional to the total labour costs. Therefore, scheme (2) is really represented in values, expressed in monetary units.

In scheme (2), the following equations are satisfied, which determine the conditions for the realization of simple reproduction:

$$\left. \begin{aligned} c_1 + v_1 + m_1 &= c_1 + c_2 + c_3 = w_1 = C \\ c_2 + v_2 + m_2 &= v_1 + v_2 + v_3 = w_2 = V \\ c_3 + v_3 + m_3 &= m_1 + m_2 + m_3 = w_3 = M \end{aligned} \right\} \quad (3)$$

Bortkiewicz is affected by a simplistic understanding of Marx's procedure for the transformation concerning the three-product scheme (2). He understands this procedure in such a way that at the first stage the general profit rate $\rho = M / (C + V) = 0.29630$ is determined, and then the total surplus value M is elementarily redistributed in the departments of the general production:

$$\left. \begin{aligned} (c_1 + v_1) + \rho(c_1 + v_1) &= u_1 \\ (c_2 + v_2) + \rho(c_2 + v_2) &= u_2 \\ (c_3 + v_3) + \rho(c_3 + v_3) &= u_3 \end{aligned} \right\} \quad (4)$$

As a result, one gets:

$$\begin{array}{lcl} \text{I. Production of means of production...} & 225c_1 + 90v_1 + 93.333p_1 & = 408.333u'_1 \\ \text{II. Production of workers' consumption} & & \\ \text{goods (wage goods).....} & 100c_2 + 120v_2 + 65.185p_2 & = 285.185u'_2 \\ \text{III. Production of capitalists' consumption goods (luxury goods)...} & 50c_3 + 90v_3 + 41.482p_3 & = 181.482u'_3 \\ \hline \text{Total annual commodity product.....} & 375C + 300V + 200M & = 875W \end{array} \quad (5)$$

The analysis of scheme (5) shows that after the transformation “according to Marx” there is a mismatch in the balance of the simple reproduction: the price of production of the means of production (u_1) is greater than the effective demand for them (C) by 8.9%, the price of production of wage goods (u_2) is lower than the effective demand for them (V) by 4.9%, and the price production of luxury goods (u_3) is lower than the effective demand for them (u_3) by 9.3%.

Marx, as indicated, planned to remove the initial premise that “the cost of producing a good is equal to the value of the goods consumed during its production”, but he did not consider necessary “to enter into this issue in detail”. Therefore, it is logical that Bortkiewicz constructs his own system of equations of the following kind:

$$\left. \begin{aligned} (1 + \rho)(c_1x + v_1y) &= (c_1 + c_2 + c_3)x \\ (1 + \rho)(c_2x + v_2y) &= (v_1 + v_2 + v_3)y \\ (1 + \rho)(c_3x + v_3y) &= (m_1 + m_2 + m_3)z \end{aligned} \right\} \quad (6)$$

The construction of system of equations (6) is based on the fact that during the transformation of the values into the production prices, the establishment of a general rate of profit (ρ) must be accompanied by a change in the value of constant production costs from (c_i) to (c_ix), and in the value of the variable production costs from (v_i) to (v_iy). Thus, we obtain three equations with four unknowns (x, y, z and ρ). In order to solve the system of equations (6), we must add either a missing fourth equation, or take one of the unknown equal to unity.

Note that Bortkiewicz defended the invariance of the unit cost for luxury goods, produced in industry III. Therefore, he assumed that $z = 1$.

“Taking a leaf out of Ricardo’s book,” writes Mark Blaug, “Bortkiewicz identified luxury goods with gold and thus ensured that money prices are expressed in terms of the labour value of gold. On applying Bortkiewicz’s solutions to the given value system, it appears that total surplus values are equal to total profits but that ‘total values’ necessarily diverge from ‘total prices’. (...) If Bortkiewicz had followed Ricardo the whole way, he would have made wage goods the ‘invariable measure of value’ by setting $p_2 = 1$ and assuming $q_2 = q_0$ [with $q_2 = c_2 / v_2$ and $q_0 = C / V$]. And, indeed, this is one of the strong assumptions required to make the sum of values come out equal to the sum of prices.” (Blaug, 1985: 233).

For $z = 1$, Bortkiewicz obtained the solution $x = 1.28$; $y = 1.0667$ and $\rho = 0.25$, and then the result of the transformation, presented in Table 1.

Table 1. Bortkiewicz-calculated prices of production (at $z = 1$)

Industry	Constant Capital	Variable Capital	Profit	Price	Rate of Surplus Value	Rate of Profit
I	288	96	96	480	1.0000000	0.250
II	128	128	64	320	0.5000000	0.250
III	64	96	40	200	0.4166667	0.250
Total	480	320	200	1000	0.6250000	0.250

It should be noted that for $y = 1$, a solution $x = 1.2$; $z = 0.9375$ and $\rho = 0.25$ would have been obtained. If the system (6) is supplemented with the second Marx’s postulate of invariance, $C + V + M = Cx + Vy + Mz$, then one obtains $x = 1.12$; $y = 0.9333$; $z = 0.875$ and $\rho = 0.25$. In this case one arrives at the production prices presented in Table 2.

Table 2. Calculation prices of production, on the basis of the Bortkiewicz system of equations provided that the sum of production prices is equal to the sum of values

Industry	Constant Capital	Variable Capital	Profit	Price	Rate of Surplus Value	Rate of Profit
I	252	84	84	420	1.0000	0.250
II	112	112	56	280	0.5000	0.250
III	56	84	35	175	0.4167	0.250
Total	420	280	175	875	0.6250	0.250

A comparison between the solutions presented in Table 1 and Table 2 shows that they are linearly dependent, since the ratio of any absolute parameter in one system to the analogous parameter in another system is expressed by the same proportionality coefficient: $k = 0.875$. But when the second postulate of Marx is fulfilled, the first postulate is not fulfilled, since the sum of the surplus values ($M = 200$) is not equal to the sum of the profits ($Mz = 175$). On this basis, Bortkiewicz said:

“...Marx simply asserts in general terms that total price equals total value. This assertion is not only unproven, but it is also false.” (Bortkiewicz, 1952: 11).

After the publication of the works by Bortkiewicz, and then by Georg von Charasoff (Charasoff, 1910), a long break occurred in the study of the transformation problem in Western literature (Desai, 1988: 296). Only in 1942 Paul Sweezy in *Theory of Capitalist Development* discussed Bortkiewicz’s results (Sweezy, 1942: 115-123), although these were mentioned earlier in the articles by Kei Shibata (Shibata, 1939). Then there were the works by Joseph Winternitz (Winternitz, 1948), Ronald Meek (Meek, 1956), Francis

Seton (Seton, 1957) and other economists, the results of which showed that the system of price equations can include only one of the Marx's postulates, and it does not matter which one. In any case, *one* of the postulates of invariance is not to be fulfilled, calling into question the consistency of the entire transformation procedure proposed by Marx. According to a number of Western economists, if the sum of values is not equal to the sum of prices, this means that at least part of the price of goods in economy has not arisen as the result of the value redistribution and so has no value nature. The latter fact raises questions about the meaning of using the value category in economic analyses.

The failure to fulfill the second Marx's postulate of invariance, i.e. the equality between aggregate profit and total surplus value, means that part of the profit has not a labour source. Such a result contradicts the Marxist theory of surplus value. With any choice of the invariance conditions, we will have to sacrifice one integral component of the Marx's doctrine (Blaug, 1985 [1962]: 232), (Borisov 2010a: 59).

As we see, in the process of studying the problem of the transformation of values into production prices by the end of the 1950s, a large number of economists became convinced that the scheme proposed by Marx for the transformation of values into prices of production was inconsistent.

This assumption contributed to the fact that many radical economists, who had ideas close to Marxism, moved into the camp of neo-Ricardians and began to criticize the Marxist theory, especially after the publication in 1960 of the work by Piero Sraffa (Sraffa, 1960).

In the mass of critical literature that appeared in the 1970's, the work of the neo-Ricardian Ian Steedman is particularly noteworthy, since he came to the conclusion that it is possible to abandon labour theory of value without compromising the results of the economic analysis (Steedman, 1977).

At the heart of his argument lay the assertion that general rate of profit and relative prices can be obtained directly from a system of equations that determines the production prices:

$$\pi = (1 + r) (\pi \mathbf{A} + \pi \mathbf{w} \mathbf{l}), \quad (7)$$

where: $\pi = (\pi_i)$ is a $1 \times n$ vector of commodity prices; $\mathbf{A} = (a_{ij})$ is a $n \times n$ matrix of specific direct cost coefficients, showing the amount of the good i needed to produce a unit of the good of type j ; $\mathbf{w} = (w_i)$ is a $n \times 1$ real wage vector, each element of which shows the amount of good i consumed per unit of used labor; $\mathbf{l} = (l_i)$ is a $1 \times n$ vector of the coefficients of specific labour intensity of the products, each element of which shows the amount of living labour necessary for the production of a unit of the good i .

In model (7), Steedman suggests that the gross output of each product is equal to one by a suitable choice of units (see Steedman, 1977: 50). It is assumed that the composition of natural wages per unit of labor is the same in all sectors: $w = w_1 = w_2 = w_3$. It follows from (7) that the profit rate (r) and the relative prices $(\pi_1 / \pi_n), (\pi_2 / \pi_n), \dots, (\pi_{n-1} / \pi_n)$ can be determined without using additional equations (Borisov, 2010b: 43).

To verify this statement, we first determine matrix of technical coefficients regarding means of production $\mathbf{A} = (a_{ij})$, as well as the vector of direct labor coefficients $\mathbf{w} \mathbf{l} = (w_i l_i)$, from the initial value-model (2): $a_{11} = c_1 / w_1 = 0.6$; $a_{12} = c_2 / w_2 = 1/3$; $a_{13} = c_{13} / w_3 = 0.25$; $w_1 l_1 = v_1 / w_1 = 0.24$; $w_2 l_2 = v_2 / w_2 = 0.4$; $w_3 l_3 = v_3 / w_3 = 0.45$.

After doing this, we can write the system of equations of production prices proposed by Steedman:

$$\left. \begin{aligned} \pi'_1 &= 0.6000\pi'_1 + 0.24\pi'_2 + r(0.6000\pi'_1 + 0.24\pi'_2) \\ \pi'_2 &= 0.3333\pi'_1 + 0.40\pi'_2 + r(0.3333\pi'_1 + 0.40\pi'_2) \\ \pi'_3 &= 0.2500\pi'_1 + 0.45\pi'_2 + r(0.2500\pi'_1 + 0.45\pi'_2) \end{aligned} \right\} \quad (8)$$

Since there is an excessive number of unknowns in the system of equations (8), it should be supplemented with an economically acceptable condition, for example, which defines the n^{th} product as a standard for measuring prices: $\pi'_n = 1$. For example, assuming $\pi'_2 = 1$, we obtain the following solution of the system of equations (8): $\pi'_1 = 1.2$; $\pi'_3 = 0.9375$ и $r = 0.25$. On this basis, we determine that $\pi'_1 / \pi'_2 = 1.2$ and $\pi'_3 / \pi'_2 = 0.9375$. Therefore, at first glance, the Steadman's production price system (8) allows one to determine relative prices and the rate of profit without using labor costs, the analysis of which becomes unnecessary. However, if you calculate these relative prices from the data in Table 1, which presents the Bortkiewicz model in production prices, we get a different result: $u_1 / u_2 = 480 / 320 = 1.5$; $u_3 / u_2 = 200 / 320 = 0.625$ and $\rho = 0.25$.

The correct result will be obtained if we replace the Leontief matrix $\mathbf{A}$ and the vector $\mathbf{wl}$ with the Mühlpfordt matrix $\mathbf{Q}$ and the corresponding vector $\mathbf{q}_2$ (see Mühlpfordt, 1895: 97): $q_{11} = (c_1 / w_1) = 0.6$; $q_{12} = (c_2 / w_1) = 0.2667$; $q_{13} = (c_{13} / w_1) = 0.1333$; $q_{21} = (v_1 / w_2) = 0.3$; $q_{22} = (v_2 / w_2) = 0.4$; $q_{23} = (v_3 / w_2) = 0.3$. In this case, the matrix $\mathbf{Q} = (q_{ij})$ shows the share of the total number of means of production needed by the j^{th} industry, and the vector $\mathbf{wl} = \mathbf{q}_2 = (q_{2j})$ shows the share of the total number of consumer goods needed by the j^{th} industry:

$$\left. \begin{aligned} \pi_1 &= 0.6000\pi_1 + 0.3\pi_2 + r(0.6000\pi_1 + 0.3\pi_2) \\ \pi_2 &= 0.2667\pi_1 + 0.4\pi_2 + r(0.2667\pi_1 + 0.4\pi_2) \\ \pi_3 &= 0.1333\pi_1 + 0.3\pi_2 + r(0.1333\pi_1 + 0.3\pi_2) \end{aligned} \right\} \quad (9)$$

Now the solution of the system of equations (9) coincides with the Bortkiewicz solution: $\pi_1 / \pi_2 = 1.5 / 1 = 1.5$ и $\pi_3 / \pi_2 = 0.625 / 1 = 0.625$ and $r = 0.25$.

Here it is appropriate to recall Engels' words that under the capitalist form of production:

The “value ... is so thoroughly well-concealed that our economists can happily deny its existence.” (Marx, 2004 [1892-5]: 462).

To reveal this hidden value, one recalls that the price system (9) is based on the information about the costs of the living labor $q_{2j} = wl_j / wL = l_j / L = v_j / V$. This is information about the vector $\mathbf{q}_2$, and also about the matrix $\mathbf{Q}$, which is quite sufficient to determine prices proportional to the total costs of the socially necessary labour, that is, values. To this end, one can apply the system of equations of Vladimir Karpovich Dmitriev (Dmitriev, 1904: 58), which in our case takes the following forms:

$$\omega = \omega \mathbf{Q} + \mathbf{q}_2. \quad (10)$$

Using the known data on $\mathbf{Q}$ and $\mathbf{q}_2$ from the numerical system of production prices (8), we obtain the following system of equations:

$$\left. \begin{aligned} \omega_1 &= 0.6000\omega_1 + 0.3 \\ \omega_2 &= 0.2667\omega_1 + 0.4 \\ \omega_3 &= 0.1333\omega_1 + 0.3 \end{aligned} \right\} \quad (11)$$

Solving the system of equations (11), we obtain the unit prices proportional to the total cost of labour-time:

$$\left. \begin{aligned} \omega_1 &= 0.3 / 0.4 = 0.75 \\ \omega_2 &= 0.2667 \cdot 0.75 + 0.4 = 0.6 \\ \omega_3 &= 0.1333 \cdot 0.75 + 0.3 = 0.4 \end{aligned} \right\} \quad (12)$$

Based on the obtained solution (12), we calculate the relative prices of $\omega_1 / \omega_2 = 0.75 / 0.6 = 1.25$ and $\omega_3 / \omega_2 = 0.4 / 0.6 = \frac{2}{3}$, and then similar prices according to the cost system of prices (2) $w_1 / w_2 = 375 / 300 = 1.25$ and $w_3 / w_2 = 200 / 300 = \frac{2}{3}$. As you can see, these prices are the same. This means that prices are really proportional to the total cost of labour-time.

Thus, the *system of production prices* (8) contains hidden information about the values of the corresponding commodity products. This means that the same technological information used to calculate the production costs allows calculating the value of goods. And vice-versa: according to the data on the value of goods, it is possible to calculate the production price system, in which one of Marx's postulates is fulfilled, as well as the postulate about the invariance of real wages absent in Marx.

But most interesting is the fact that *the system of production prices contains information on the general rate of surplus value*, which for the cost system (2) is $m' = M / V = w_3 / w_2 = 200 : 300 = 0.6667$. For the cost system (10), this ratio is also equal to $m' = \omega_3 / \omega_2 = 0.4 / 0.6 = 0.6667$. This fact refutes the findings of Tugan-Baranovsky, who wrote that with the transition from values to production prices or from production prices to values:

"All the distribution relations vary, depending on whether we express them in monetary prices or labour values. Thus, for example, the total variable capital (wages) is equal in the first scheme to $200/625 = 32\%$ of the total social product, and by its labour value it reaches $300/875 = 34\%$ of the value of the product. The percentage of profit, at a money price, was 25%, and in its labour value, the social profit reaches $200/625$, almost 30% of social capital." (Tugan, 2003 [1905]: 157-158).

Tugan-Baranovsky¹ did not take into account that after a direct (or inverse) transformation of values (or prices), when the postulate of the invariance of real wages is introduced, there is an increase (or decrease) in the wages and the distributed proportion also changes, but changes *nominally*. Indeed, a comparison of the cost scheme (2) with the scheme in production prices (see Table 1) shows that the monetary expression of the variable capital increased by 6.67%, and the price of constant capital by 28%, but the workers receive the same amount consumable goods.

However, Tugan-Baranovsky skipped such an analysis, since he expressed the value in natural, rather than monetary, units which does not correspond to the Marxist methodology. After all, in the first volume of *Capital*, Marx explained:

¹ "The assertion of Marx," wrote Tugan-Baranovsky, " 'that all the deviations of the prices of production from the values mutually cancel each other' is erroneous in the sense that such a mutual cancelation of the deviations takes place only in relation to the aggregate of the total product as a whole, but not in relation to its divisions in total capital and profit, once that the general percentage of profit is established." (Tugan, 2003 [1905]: 158).

“Magnitude of value expresses a relation of social production, it expresses the connection that necessarily exists between a certain article and the portion of the total labour time of society required to produce it. As soon as magnitude of value is converted into price, the above necessary relation takes the shape of a more or less accidental exchange ratio between a single commodity and another, the money commodity.” (Marx, 1996 [1867]: 111-112).

Marx also pointed out the following:

“The distinction between price and value, between the commodity as measured by the labour time of which it is the product, and the product of the labour time for which it is exchanged, this distinction demands a third commodity as a measure, in which the real exchange value of the commodity is expressed. *Because price does not equal value, the element determining value, labour time, cannot be the element in which prices are expressed. For labour time would have to express itself at once as the determining and the non-determining element, as the equivalent and the non-equivalent of itself.* Because labour time as a measure of value exists only ideally, it cannot serve as the material for the comparison of prices.” (Marx, 1986 [1857-61]: 77).

However, on this list of distortions of Marx’s theory Tugan-Baranovsky, uncritically received by Bortkiewicz, does not end there. Tugan-Baranovsky, having set the goal to definitively refute Marx’s theory of the surplus value (see Tugan, 2003 [1905]: 155), admits the following distortions of the Marxian theory of formation of the general rate of profit (the average rate of profit) and transformation of the value of goods into the price of production.

First, Tugan-Baranovsky interprets the second Marx’s postulate of invariance (the sum of the prices of production of the entire social product must be equal to the sum of its value) in the sense that by social product he means the aggregate social product, while Marx understood this product as the end (final) social product. In particular, in the 9th chapter of volume III of the *Capital*, Marx unambiguously explains that he is considering the amount of deviations of profits from surplus values only for those products that are end, and not intermediate products.

“Considering the calculation as a whole,” writes Marx, “we see that since the profits of one sphere of production pass into the cost price of another, they are therefore included in the calculation as constituents of the total price of the end product, and so cannot appear a second time on the profit side. If any do appear on this side, however, then only because the commodity in question is itself an ultimate product, whose price of production does not pass into the cost price of some other commodity.” (Marx, 1997 [1885]: 159).

From this it follows that in the tables of the 9th chapter of volume III of the *Capital*, Marx presents the final products of the five independent spheres of production that fabricate goods I-V. Marx explains that:

“The aggregate price of the commodities I to V would therefore equal their aggregate value, i.e., the sum of the cost prices I to V plus the sum of the surplus values, or profits, produced in I to V. It would hence actually be the money expression of the total quantity of past and newly added labour incorporated in commodities I to V. And in the same way the sum of the prices of production of all commodities produced in society — the totality of all branches of production — is equal to the sum of their values.” (Marx, 1997[1885]: 158-159).

This premise means that the total amount of the annual expenditures of living labour used in the production of final and intermediate goods, in the transformation of the value of goods into their production price, does not change, since it is a technologically predetermined quantity. In particular, it follows from system (10) that for simple reproduction:

$$q_{21} + q_{22} + q_{23} = \omega_2 + \omega_3. \quad (13)$$

The left-hand side of equation (13) shows the total amount of labour input, which is the source of the value newly created in a year. In this case, the costs of living labor (q_{21}) are embodied in intermediate

goods (i.e. in constant capital), and the costs of living labor (q_{21}) and (q_{23}) in the relevant final goods. The right side of equation (13) simultaneously reflects the sum of the total newly created value and the fact that, in simple reproduction, all the living labour of the society is embodied in the final goods, i.e. in commodities for workers (ω_2) and capitalists (ω_3).

In this respect, the following remark by Marx is of fundamental importance:

“It is quite correct to say that the component parts of commodities which make up the constant capital, like any other commodity value, may be reduced to portions of value which resolve themselves for the producers and the owners of the means of production into wages, profit and rent. This is merely a capitalist form of expression for the fact that all commodity value is but the measure of the socially necessary labour contained in a commodity. But it has already been shown in Book I that this nowise prevents the commodity product of any capital from being split into separate parts, of which one represents exclusively the constant portion of capital, another the variable portion of capital, and a third solely surplus value.” (Marx, 1997 [1885]: 838).

If this is so, then for the system of value equations (10) the following equality holds:

$$q_{12}\omega_1 + q_{13}\omega_3 = q_{21}. \quad (14)$$

Indeed, equality (14) is satisfied, since $0.2667 \cdot 0.75 + 0.1333 \cdot 0.75 = 0.3 = q_{21}$. Thus, there is no doubt that by Marx, as social product, it should be understood the *final social product*, and not the aggregate social product, as is commonly believed in Western economic literature. In volume II of the *Capital* there is Marx's explanation of what should be understood as *social product*:

“One point herein is correct: that the matter presents itself differently in the movement of social capital, i. e., of the totality of individual capitals, from the way it presents itself for each individual capital considered separately, hence from the standpoint of each individual capitalist. For the latter the value of commodities resolves itself into 1) a constant element (a fourth one, as Adam Smith says), and 2) the sum of wages and surplus value, or wages, profit, and ground rent. But from the point of view of society the fourth element of Adam Smith, the constant capital value, disappears.” (Marx, 1997 [1985]: 383).

This means that the social product consists in the sum $(V + M)$, and not $(C + V + M)$. But this circumstance does not prevent the possibility of depicting individual final products included in $(V + M)$ as a sum of $(c_i + v_i + m_i)$.

Secondly, Tugan-Baranovsky, through the use of the rigid design of the three-branch simple reproduction scheme (2), implicitly introduced an *additional postulate of invariance*: the assertion that real wages do not change when the value of goods is converted into their production price. But at the stage of “of the original conversion of values into production prices and the original establishment of the general rate of profit”, Marx does not have “variations in the rate of a surplus value corresponding to a rise or a fall of wages” which would maintain the level of the real wages (see Marx, 1991 [1861–3]: 261).

“In our consideration of the transformation of surplus value into profit,” wrote Marx, “we assumed that wages do not fall, but remain constant, because there we had to investigate the fluctuations in the rate of profit, independent of the changes in the rate of surplus value.” (Marx, 1998 [1894]: 844–845).

The consequence of this approach of Marx was that after the initial transformation of values into production prices presented in Chapter 9 of Volume III of the *Capital*, there is a slight change in the level of real wages. Here is how Marx writes about this fact:

“As for the variable capital, the average daily wage is indeed always equal to the value produced in the number of hours the labourer must work to produce the necessities of life. But this number of hours is in its turn obscured by the deviation of the prices of production of the necessities of life from their values. However, this

always resolves itself to one commodity receiving (too little of the surplus value while another receives too much, so that the deviations from the value which are embodied in the prices of production compensate one another.” (Marx, 1998 [1894]: 160).

Nevertheless, the mutual compensation of deviations from values, which are embodied in the prices of production of wage goods, is not complete:

“Suppose, the average composition is $80_c + 20_v$. Now, it is possible that in the actual capitals of this composition 80_c may be greater or smaller than the value of c , i. e., the constant capital, because this c may be made up of commodities whose price of production differs from their value. In the same way, 20_v might diverge from its value if the consumption of the wage includes commodities whose price of production diverges from their value; in which case the labourer would work a longer, or shorter, time to buy them back (to replace them) and would thus perform more, or less, necessary labour than would be required if the price of production of such necessities of life coincided with their value.” (Marx, 1998 [1894]: 204).

In the *Grundrisse*, having considered the simplest examples of the formation of a general rate of profit and its effect on the wages of the worker, Marx comes to the following important conclusion:

“For the worker, therefore, all three cases are possible: his gain or loss by the operation [the evening-up of profits] could=zero; the operation could depreciate his necessary wage so that it no longer suffices, hence depress it below the necessary minimum; lastly, it could create for him a surplus wage, which would amount to an extremely small share of his own surplus labour.” (Marx, 1986 [1857-61]: 366).

Let us consider the transformation of values into production prices under those postulates of invariance that Marx himself introduced. To this end, we use the simple reproduction scheme (2) as the basis of the transformation and we also assume that during the transformation the sum of the profits of all the different spheres (branches) of production must be equal to the sum of surplus values, and the sum of the prices of production of the final product is equal to the sum of its value. Under such assumptions, it is sufficient to solve the following system of equations with an unknown index of the price of the constant capital (x) and a new unknown total profit rate (r) for transforming the values of commodity products of the scheme (2) into appropriate prices:

$$\left. \begin{aligned} w_1 &= (c_1x + v_1)(1+r) \\ r &= M / (w_1x + V) \end{aligned} \right\} \quad (15)$$

Solving the system of equations (15) using data w_1, c_1, v_1, V and M from the scheme (2), we obtain $x = 1.24339811320566$ and $r = 0.261003144657233$. The original version of the transformation of values into production prices is shown in Table 3.

An analysis of the results of the transformation of values into the production prices shows that after the establishment of the general rate of profit, a direct modification of the value of constant capital takes place, the price of which increases by 1.243 times, and also an indirect modification of variable capital, which remains nominally unchanged ($V = 300$) and can use only a part of its previously owned means of consumption (300 out of 308.11327, i.e. about 97.4%). This is due to the fact that the production price of consumable goods is higher than its value by 2.6%.

The opposite result is achieved for the capitalists. In the example under consideration, at the production prices, the capitalists will be able to consume not only all their luxury goods, but also a part of the consumable goods worth 8.11327 monetary units. Thus, there is a certain change in the real incomes of workers and capitalists, and this change is the lower the more insignificant is difference in the organic composition of capital in the various branches, and the larger is the number of consumable goods

produced in the economic system, since here the action of the law of the large numbers begins to manifest, leading to a more complete mutual repayment of positive and negative deviations of prices from values.

Table 3. The calculation of the production prices in full accordance with the two postulates of invariance of the original transformation by Marx²

<i>Industry</i>	<i>Constant Capital</i> c'	<i>Variable Capital</i> v	<i>Profit</i> p'	<i>Price</i> $u' = c' + v + p'$	<i>Rate of Surplus Value</i> $m' = p' / v$	<i>Rate of Profit</i> r
I	279.76458	90	96.50972	466.27429	1.07233	0.261003
II	124.33981	120	63.77346	308.11327	0.53145	0.261003
III	62.16991	90	39.71682	191.88673	0.44130	0.261003
<i>Total</i>	466.27429	300	200	966.27429	0.66667	0.261003

The critics of Marx consider the change in real wages during the initial transformation not otherwise than the collapse of Marx's theory, i.e. as a situation that from a logical point of view is unsatisfactory. However, it should be kept in mind that capitalists consume the same necessary commodities as workers.

For example, the increase in their real income by reducing the price of luxury goods in comparison with their value can be spent either on the direct consumption of additional commodities, or on additional luxury goods, if we take into account the foreign trade of surplus commodities needed for luxury goods.

Let us note that Marx understood this contradiction and, in addition to indicate the probability of mutual repayment of the deviations from value (consisting in the production prices of consumable goods for workers), formulated a *second approach* to his solution. He wrote:

“Suppose that the general rate of profit, and therefore the average profit, are expressed by money value greater than the money value of the actual average surplus value. So far as the capitalists are concerned, it is then immaterial whether they reciprocally charge 10 or 15% profit. Neither of these percentages covers more actual commodity value than the other, since the overcharge in money is mutual. As for the labourer (the assumption being that he receives his normal wage and the rise in the average profit does not therefore imply an actual deduction from his wage, i.e., something entirely different from the normal surplus value of the capitalist), *the rise in commodity prices caused by an increase of the average profit must correspond to the rise of the money expression of the variable capital*. Such a general nominal increase in the rate of profit and the average profit above the limit provided by the ratio of the actual surplus value to the total invested capital is not, in effect, possible without causing an increase in wages, and also an increase in the prices of commodities forming the constant capital. The reverse is true in case of a reduction.” (italics mine — V.K.), (Marx, 1998 [1894]: 178-179).

What does Marx's explanation mean in relation to the result of the initial transformation of value into production price given in Table 3? So, from the table it follows that the average profit is 200 monetary units and is higher than the real average surplus value in monetary terms, amounting to 191.88673. At the same time, workers do not receive their normal wages, therefore an increase in the average profit means a real deduction from wages, that is, the profit is too high in spite of the fact that the nominal rate of surplus value remained at the level of 0.667.

Thus, after the initial transformation of the values into the production prices, the results of which are presented in Table 3, there was a general nominal increase in the rate of profit and the average profit

² The results of this calculation are consistent with those of Mark Glick and Hans Ehrbar (Glick and Ehrbar, 1987) and Dong-Min Rieu (Rieu, 2006: 265), performed in accordance with the *New Interpretation*, sponsored by Duncan K. Foley (Foley, 1982), Gérard Duménil (Duménil, 1983-4), etc.

above the level determined by the ratio of the real surplus value to all the advanced capital. In other words, the rate of profit $r = 0.261003$ is too high and this should cause an increase in the real wages, as well as an increase in the prices of goods forming a constant capital. An increase in the prices of goods constituting a constant capital is required to balance the growth of the monetary wage in order to equalize the general rate of profit at a lower level.

In order to see the indicated changes, we solve the following system of equations, in which the initial parameters must be taken from Table 3:

$$\left. \begin{aligned} u_1 x &= (c'_1 x' + v_1 y)(1 + r') \\ u_2 y &= (c'_2 x' + v_2 y)(1 + r') \\ u_3 z &= (c'_3 x' + v_3 y)(1 + r') \end{aligned} \right\} \quad (16)$$

Here the indices of change in the prices of constant and variable capital, respectively (x') and (y), and also the new average rate of profit (r'), are unknown. Using the data in Table 3 and solving the corresponding system of equations (16), we obtain: $x' = 1.02943698112906$; $y = 1.0667$ and $r' = 0.25$. After this, it is simply enough to transform the data of Table 3. The result of the transformation is shown in Table 4.

Table 4. The result of the final transformation of values into production prices in accordance with the methodology of Marx

<i>Industry</i>	<i>Constant Capital</i> $c'x' = c''$	<i>Variable Capital</i> $vy = v''$	<i>Profit</i> p''	<i>Price</i> u''	<i>Rate of Surplus Value</i> m''	<i>Rate of Profit</i> r'
I	288	96	96	480	1.00000	0.25
II	128	128	64	320	0.50000	0.25
III	64	96	40	200	0.41667	0.25
<i>Total</i>	480	320	200	1000	0.62500	0.25

The analysis of the data of Table 4 shows that after the final transformation all the balance conditions for the realization of simple reproduction are fulfilled:

$\sum_i c''_i = u''_1$, $\sum_i v''_i = u''_2$ and $\sum_i p''_i = u''_3$. What is important is that all those changes in the parameters about which Marx wrote are actually confirmed, namely:

- 1) the nominally overestimated rate of profit, which caused a fall in the real wages, decreases from 0.261003 to 0.25;
- 2) there is an increase in the wages from 300 to 320, that is, to a level at which money wages correspond again to real wages;
- 3) the price of the constant capital increases from 466.27429 to 480.

No less important for understanding the problem of transformation is that Table 4 completely reproduces the result of Bortkiewicz's transformation, presented in Table 1. But now it becomes clear that the allegation by Bortkiewicz to Marx of using the erroneous invariant postulate — the aggregate price is equal to the aggregate value — is devoid of logical grounds. This postulate refers to the initial

transformation and, in addition, to the totality of commodity products forming the final social product, and not the gross social product.

It is quite obvious that the introduction of a new postulate on the invariance of the real wages makes the equality between the aggregate price of production and the aggregate value unnecessary, since the monetary expression of real wages changes. But Marx never insisted that this postulate of equality between the aggregate price of production and the total value should be fulfilled at the stage of the final transformation of values into production prices. It should be borne in mind that at final prices of production, the amount of wages nominally deviates from the real value of variable capital. Therefore, it is impossible to accurately calculate the real rate of surplus value (M/V) with the help of data from the numerical model presented in Table 4. To calculate the real rate of surplus value, it is necessary *to inverse the transformation of production prices into values*³. For the model in question, presented in Table 4, the inverse transformation algorithm reduces to finding unknown parameters J_c , J_v and m' from the following system of equations:

$$\left. \begin{aligned} c_1''J_c + v_1''J_v + v_1''J_v m' &= u_1''J_c \\ c_2''J_c + v_2''J_v + v_2''J_v m' &= u_2''J_v \\ c_3''J_c + v_3''J_v + v_3''J_v m' &= u_3'' \end{aligned} \right\} \quad (17)$$

where m' is the real general rate of surplus value, determined by the formula: $m' = \Sigma p_i'' / (J_v \Sigma v_i'')$; J_c is the index of change in the price of production of constant capital after the transition to its value; J_v is the index of the change in the price of production of consumable goods for workers after the transition to their value.

Given that: $J_v m' = \Sigma p_i'' / \Sigma v_i'' = P'' / V''$, the system of equations (17) is simplified:

$$\left. \begin{aligned} c_1''J_c + v_1''J_v + v_1''\frac{P''}{V''} &= u_1''J_c \\ c_2''J_c + v_2''J_v + v_2''\frac{P''}{V''} &= u_2''J_v \\ c_3''J_c + v_3''J_v + v_3''\frac{P''}{V''} &= u_3'' \end{aligned} \right\} \quad (18)$$

Using the first and third equation of system (18), we obtain the formulas for determining the unknowns J_c and J_v :

$$J_v = \frac{(c_1'' - u_1'')(P'' - v_3''\frac{P''}{V''}) + c_3''v_1''\frac{P''}{V''}}{(c_1'' - u_1'')v_3'' - v_1''c_3''}; \quad (19)$$

$$J_c = \frac{-v_1''P''}{(c_1'' - u_1'')v_3'' - v_1''c_3''}. \quad (20)$$

³ “The problem of inverse transformation”, writes Foley, “poses the question before us: how based on data on the implementation of surplus value, conclude where it is produced?” (Foley, 2017: 273).

Substituting in equations (19) and (20) the known parameters of the numerical model from Table 4 and solving it with respect to the unknowns, we obtain: $J_c = 0.78125$ and $J_v = 0.9375$. After this, the actual general rate of surplus value is calculated: $m' = P'' / (V''J_v) = [200 / (320 \times 0.9375)] \times 100\% = 66.67\%$. With the help of the known quantities J_c , J_v and m' , the model with the final production prices (see Table 4) is transformed into the original value model, represented in the form of the system of equations (2).

Thus, between the sector value model and the same model with the final production prices, there is an unconditional internal relationship. The link between the system of values and the system of production prices is the amount of profit equal to the sum of surplus value and, of course, the unchanged technological coefficients.

As can be seen from equations (18), for an inverse transformation of the prices of production of goods into their values, no preliminary information is required on the real general rate of surplus value. And, nevertheless, the prices of production can be transformed into values so to determine the real general rate of surplus value. This means that the level of the general rate of the real surplus product is determined by the production price system, as it was also demonstrated by the algorithms of Steedman or Dmitriev.

Therefore, the general rate of surplus value, admitted by Marx as a theoretical simplification, is, in fact, not only a historical prerequisite of the system of production prices established as a result of the mutual competition of capitals, but is also a practical consequence of the existence of this system of prices. *The production price system contains information on the overall degree of labour exploitation.*⁴

The practical usefulness of the theory and the methods of direct and inverse transformation of value into price is the following. They allow us to justify the methods of measurement of efficiency in social production (large-scale national economic investment projects) on the basis of values, and not only on the basis of production prices alone, which do not allow us to reveal the actual effectiveness of these projects on the ground of measuring social labour productivity (Marx, 1998 [1894]: 259-261).

The results of the study of the measured properties of value prices and production prices are given in Table 5 and Table 6.

Table 5. Comparison of the two options of the functioning of economy when using value prices

<i>Industry</i>	<i>Constant Capital</i>	<i>Variable Capital</i>	<i>Profit</i>	<i>Price</i>	<i>Rate of surplus value</i>	<i>Rate of profit</i>
a) basic production option						
I	225	90	60	375	0.66667	0.19048
II	100	120	80	300	0.66667	0.36364
III	50	90	60	200	0.66667	0.42857
<i>Total</i>	375	300	200	875	0.66667	0.29630
b) production option, when in the second industry the costs of the constant capital increase by 1.25 times and the costs of the variable capital are reduced by 1.2 times, at constant value prices						
I	262.5	105	70	437.5	0.66667	0.19048
II	125	100	75	300	0.75000	0.33333
III	50	90	60	200	0.66667	0.42857
<i>Total</i>	437.5	295	205	937.5	0.69492	0.27986

⁴ Now we can confidently refute the statement of Paul Samuelson's and write that after the inverse transformation, the total profit is allocated by the value system according to where it was really produced (see Samuelson, 1971: 417).

The comparison between option b) and option a) shows that, in terms of productivity growth, option b) is still more effective in this case. In particular, at a constant production price for the goods of the second industry, the rate of surplus value in option b) increases from 0.5 to 0.6.

The comparison of the option b) and the option a), presented in Table 5, shows that in terms of productivity growth the option b) is more effective. In particular, with the price of the second industry remaining unchanged, the rate of surplus value in the option b) increases from 0.6667 to 0.75.

If according to the basic production option a) the productivity of the social labour for the final product was $(300w_2 + 200w_3) / 300V = 1.6667$, then, according to the option b), it increased to $(320w_2 + 200w_3) / 295V = 1.6949$, or by 1.7%. Consequently, option b) is actually more effective than option a).

Now we consider an analogous model of reproduction based on production prices and reported in Table 6. If according to the basic production option a) the productivity of the social labour in the final product was $(320w_2 + 200w_3) / 320V = 1.625$, then according to option b) it increased to $(320w_2 + 200w_3) / 314.667V = 1.65254$, or 1.7%. Consequently, option b) is actually more effective than a) in terms of productivity of social labour.

Table 6. Comparison of the two options of the functioning of economy when using production prices

<i>Industry</i>	<i>Constant Capital</i>	<i>Variable Capital</i>	<i>Profit</i>	<i>Price</i>	<i>Rate of surplus value</i>	<i>Rate of profit</i>
a) basic production option						
I	288	96	96	480	1.00000	0.25000
II	128	128	64	320	0.50000	0.25000
III	64	96	40	200	0.41667	0.25000
<i>Total</i>	480	320	200	1000	0.62500	0.25000
b) production option, when in the second industry the costs of the constant capital increase by 1.25 times and the costs of the variable capital are reduced by 1.2 times, at constant production prices						
I	336	112	112	560	1.00000	0.25000
II	160	106.66667	53.3333	320	0.50000	0.20000
III	64	96	40	200	0.41667	0.25000
<i>Total</i>	560	314.66667	200.3333	1080	0.65254	0.23467

But in conditions of production prices and competition of capitals, instead of the criterion for the greatest growth in the productivity of social labour, as it is well known, the local criterion for the growth of the rate of profit is applied.

The analysis of the data of Table 6 shows that the rate of profit decreased from 25% in the basic option a) to 20% in the basic option b). Consequently, option b), in terms of inefficient rate of profit, leads to a reduction in the overall payback period of capital, and it is unlikely that anyone would implement a less efficient investment project, especially if it is a large-scale national economic project. The decrease in the total amount of labour entering the commodity would seem to serve as an essential sign of an increase in the productive power of labour in any general condition of production.

However, as Marx wrote:

“The law of the increased productive power of labour is not, therefore, absolutely valid for capital. So far as capital is concerned, this productive power does not increase through a saving in living labour in general, but only through a saving in the *paid* portion of living labour, as compared to labour expended in the past, as we have already indicated in passing in Book I (Kap. XIII, 2, S. 409/398).a Here the capitalist mode of production is beset with another contradiction. Its historical mission is unconstrained development in geometrical

progression of the productivity of human labour. It goes back on its mission whenever, as here, it checks the development of productivity.” (Marx, 1998[1894]: 261).

We note that this contradiction, which actually takes place under capitalism, can be resolved in those countries where a state regulation of the market economy is carried out. As a last resort, nothing prevents the effectiveness of large investment projects done by the state to be assessed using appropriate value categories (see Kalyuzhnyi, 2010). Thus, the analysis of the labour costs is not at all superfluous, as Ian Steedman argued.

Conclusions

So, it can be argued that such a prolonged existence of the transformation problem is caused, on the one hand, by an inattentive reading of the works of Marx, and on the other hand - there could not have been without “Furies of private interest” (Marx, 1996 [1867]: 10). In addition, many people wanted to surpass the great scientist that Marx was.

This article shows that the relative production prices can indeed be determined without the use of values, based only on: the technological coefficients, the coefficients of consumption of the means of production, and the labour per unit of output, as established by Steedman.

However, the values can be determined on a similar basis using the system of equations of the total labour costs by Dmitriev. This means that the production price system contains in a hidden form information about values prices, directly proportional to the total labour costs.

The article shows that the price system also contains information on the real rate of surplus value, which differs from the apparent average rate of surplus value in conditions of production prices. This means that it is possible to develop a method *for the inverse transformation of the price of production of goods into value*. This method is shown for the first time in this article. It is also established that, in full accordance with Marx’s methodology, it is necessary to distinguish, first, the stage of the original transformation of values into production prices, which is based on the two well-known Marx’s postulates of invariance. In the article, it is proved that as Marx’s ‘social product’ meant the final social product, and not the aggregate social product, as most of the Marx’s opponents still believe⁵. The realization of this feature leads to a change in the real wages of workers, which, according to Marx, cannot be significant, taking into account the mutual cancellation of the deviations from the value of the numerous commodities.

Secondly, it is necessary to distinguish the stage of the final transformation of values into production prices, at which the real wages of workers are restored through a corresponding change in the level of the prices of consumable goods for workers, and constant capital. In this case the equality of the total surplus value and total profit holds, but the second postulate of invariance (the sum of the prices of production of the entire social product must be equal to the sum of its values) is replaced by the postulate of the invariance of the real wages.

The paradox is that the solution obtained by Bortkiewicz completely corresponds the results of the final transformation, which Marx substantiated. However, neither Bortkiewicz nor his followers understood that in the final transformation only one of Marx’s postulates should be fulfilled, since the second postulate is replaced by a new postulate of the invariance of the real wages. For example, if the amount

⁵ “The entire value portion of the annual product, then, which the labourer creates in the course of the year, is expressed in the annual value sum of the three revenues, the value of wages, profit, and rent. Evidently, therefore, *the value of the constant portion of capital is not reproduced in the annually created value of product*, for the wages are only equal to the value of the variable portion of capital advanced in production, and rent and profit are only equal to the surplus value, the excess of value produced above the total value of advanced capital, which equals the value of the constant capital plus the value of the variable capital.” (italics mine — V.K.), (Marx, 1998 [1894]: 821).

of profit is equal to the sum of the surplus value of all the spheres of production, the sum of the production prices of the entire social product must deviate from the sum of its values. However, this deviation is nominal in the same way as the GDP at current prices deviates from the GDP at constant prices.

The algorithms of a transformation of price values into production prices, and production prices into price values, considered in this paper allow us to substantiate new methods for measuring the effectiveness of the social production (large-scale national economic investment projects) based on measuring the social productivity of labour.

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